Session 6
Takeoff Performance

1.0 Definitions

Coefficient of friction (μ) - A measure of the resistance to movement of two surfaces which are in contact.

Takeoff distance - The distance required to accelerate an aircraft to takeoff speed

Takeoff speed - The speed where the wings of an aircraft generate enough lift to just equal the weight of the aircraft.

2.0 Introduction

Previous sessions have applied Newton's Laws to the forces of flight; weight, lift, drag and thrust. In this session, those same laws are applied to determine the takeoff performance of an aircraft. The primary consideration when analyzing takeoff performance is measuring the distance required to become airborne. In other words, determining the distance required to accelerate a stopped aircraft to an airspeed where the wings can generate enough lift to cause the aircraft to become airborne. This evaluation begins by considering the four forces associated with the test aircraft.

****START VIDEO****

3.0 Test Aircraft Description

Whenever a test report is written about an aircraft, the first thing given is a description of the test aircraft. In the video, the test aircraft is the Aermacchi MB-326 Impala jet trainer.

Figure 6.1 Impala Jet Trainer

This aircraft is used by a number of air forces throughout the world and is used as a flight test trainer at the National Test Pilot School. The aircraft has the following features:

1. Weight: 7887 pounds (includes pilots and fuel)
2. Wing Area: 205.33 square feet
3. Maximum Lift Coefficient: $C_{L\,max} = 1.51$
4. Static maximum thrust at Mojave Airport: 2200 pounds
5. Drag coefficient: $C_D = 0.06$

NOTE:

The air density (ρ) at Mojave Airport is usually about 0.0022 slugs per cubic foot.

This information is necessary to calculate the takeoff performance.

4.0 Determining the Takeoff Speed

The ***minimum lift*** required for flight, that is to just become airborne, ***occurs at a speed where the lift and weight just become equal.***

NOTE:

At speeds above that where the lift and weight just become equal, the aircraft will be able to climb or accelerate. At speeds below this there will not be enough lift generated to become airborne.

To calculate the takeoff speed for the Impala, begin with the lift equation.

$$w = L = \tfrac{1}{2}\rho V^2 S C_L \qquad (6.1)$$

Taking the numbers from the aircraft description above and rearranging Equation 6.1, the speed to just become airborne, that is the takeoff speed, is found by the following:

$$V = \sqrt{\frac{2w}{\rho S C_L}}$$

$$V = \sqrt{\frac{2(7887\,lbs)}{\left(0.002\frac{slugs}{ft^3}\right)(205.33\,ft^2)(1.51)}}$$

V = 152.1 ft/sec (or 90 knots)

To propel the aircraft forward to achieve this speed, the aircraft must have enough thrust to overcome the drag.

5.0 Determining the Drag

Session 4 demonstrated that drag varies with speed. Recall the drag equation is written as:

$$D = \tfrac{1}{2}\rho V^2 S C_D \qquad (6.2)$$

However, as the aircraft accelerates for takeoff, the speed is constantly changing so what speed is entered into equation 6.2? Through experience, engineers have learned that if 70% of the takeoff speed is used to calculate the drag during takeoff, the results are very close to the actual drag. The takeoff speed for the Impala has just been calculated at 152.1 feet per second. Then to calculate the drag, a value of 106.5 feet per second (70% of 152.1) is used in equation 6.2. Using the values for density, drag coefficient and wing area, the drag which must be overcome during the takeoff is:

$$D = \tfrac{1}{2}\rho V^2 S C_D$$

$$D = \tfrac{1}{2}\left(0.0022\tfrac{slugs}{ft^3}\right)\left(106.5\tfrac{ft}{\sec}\right)^2(0.06)$$

$$D = 153.7 \text{ lbs}$$

This drag must be subtracted from the total thrust, since these forces act in opposite directions. Additionally, since the thrust and drag are not equal, the unbalanced force (thrust) will cause the aircraft to accelerate. To determine the rate of acceleration, Newton's second law is used.

6.0 Determining the Acceleration

To calculate the acceleration rate from the $F = ma$ equation, the forces must be inserted.

$$F = T - D = ma$$

This equation can be rearranged to solve for the acceleration;

$$a = \frac{T - D}{m} \qquad (6.3)$$

Recall from session 2 the mass is the aircraft's weight divided by the acceleration of gravity:

$$m = w/g$$

Then inserting this into equation 6.3 gives;

$$a = \frac{g(T - D)}{w} \qquad (6.4)$$

Using the appropriate values from the test aircraft description:

$$\boldsymbol{a} = \frac{\left(32.2\tfrac{ft}{\sec^2}\right)(2200lbs - 153.7lbs)}{(7887lbs)}$$

$$a = 8.35 \text{ ft/sec}^2$$

This acceleration rate can be used as the slope of a straight line to construct a graphic plot of velocity (in feet/second) versus time (in seconds). Assuming this acceleration rate is constant, at the end of one second, the velocity is 8.35 ft/sec; at the end of 2 seconds, the velocity is 2 times 8.35, or 16.7 ft/sec. This method can be continued for as long as desired, but since the velocity at takeoff has already been calculated from equation 6.1 and found to be 152.1 ft/sec, then if the speed is divided by the acceleration rate, the time required to reach that speed can be determined.

$$time = \frac{\text{takeoff speed}}{\text{acceleration rate}}$$

$$time = \frac{152.1\tfrac{ft}{\sec}}{8.35\tfrac{ft}{\sec^2}}$$

$$time = 18.21 \text{ sec}$$

To construct the graph, perform the following:

- at time zero the speed is zero, then place a dot at the origin
- place another dot at the point where the speed is 152.1 ft/sec and the time is 18.21 sec
- connect the two dots

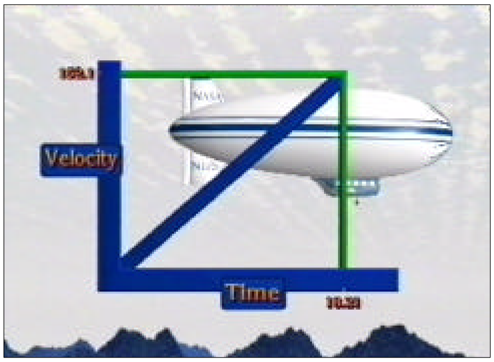

Figure 6.2 Plot of Velocity vs. Time

The predicted takeoff distance is simply the area under the line which was just drawn.

7.0 Determining the Takeoff Distance

Since a, the acceleration, is assumed to be constant, the resulting slope is a straight line. It can be seen in Figure 6.1 that by locating the point on the line associated with the takeoff speed then dropping straight down to the times axis, the area under this curve is equal to the area of a right triangle. The equation for the area of a right triangle is:

$$area = \tfrac{1}{2}(base \times height) = takeoff\ distance$$

The base of the acceleration plot is the "time" axis and the height is the "velocity axis. By substituting into equation the appropriate values from the curve, the area is found to be:

$$area = \tfrac{1}{2}[(18.21\,\text{sec})(152.1\tfrac{ft}{\text{sec}})]$$

$$area = takeoff\ distance = 1384.9 \text{ feet}$$

Then the calculated takeoff distance is approximately 1385 feet. The actual takeoff distance exceeded the predicted distance by a considerable amount.

Figure 6.3 Takeoff Distance Determination from Chase Aircraft

After the flight, the test team realized they had forgotten to account for the rolling friction of the aircraft.

8.0 Rolling Friction

NOTE:

Further explanation of rolling friction can be found in Session 4's Operational Supplement.

It takes much less force to push a hockey puck across the ice than it does to push that same puck across asphalt. This is because there is less friction to resist the movement of the puck when it is pushed across the ice. Air hockey games blow air up through holes in the table surface so the puck then rides on a cushion of air. This eliminates almost all the friction allowing the puck to move with very little force applied. Each type of surface has certain friction factor called the "coefficient of friction" and given the Greek symbol "μ". This coefficient has been determined experimentally for each type of surface and the values placed in a table. In this application, the coefficient for rubber tires on a concrete runway is approximately 0.05.

The force required to overcome friction and move an object depends on the object's weight and the surface on which it rests as shown in the following equation:

$$Friction = \mu w \qquad (6.5)$$

Then for the Impala on a concrete runway, the friction force which must be overcome before movement can begin is:

$$Friction = (0.05)(7887 \text{ lbs})$$

$$Friction = 394.4 \text{ lbs}$$

Since the rolling friction resists movement, it actually acts in the same direction as the drag and must therefore be subtracted from the thrust. Including the friction into equation 6.4, the new acceleration can be calculated:

$$a = \frac{g(T - D - Friction)}{w} \qquad (6.6)$$

$$a = \frac{\left(32.2\frac{ft}{\sec^2}\right)(2200lbs - 153.7lbs - 394.4lbs)}{7887lbs}$$

$$a = 6.74 \text{ ft/sec}^2$$

This acceleration rate is significantly slower than the 8.35 ft/sec^2 rate determined earlier. Using this new acceleration rate as the slope, a new curve can be generated in the same manner as the previous acceleration curve. The takeoff speed will remain the same since the speed depends on the aircraft weight, which hasn't changed. The final point then corresponds to a time of 22.6 sec.

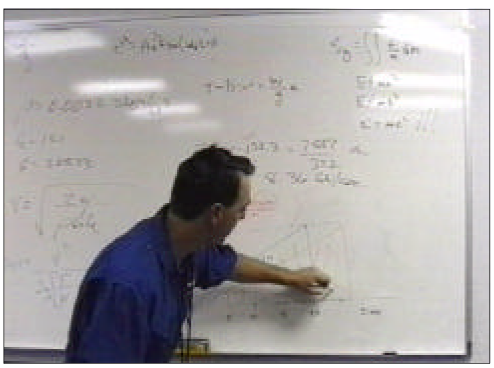

Figure 6.4 Plot of Speed vs. Velocity

NOTE:

The time required to accelerate to the takeoff speed can also be found using the following relation:

$$time = \frac{\text{velocity}}{\text{acceleration rate}}$$

$$time = \frac{(152.1\frac{ft}{\sec})}{\left(6.74\frac{ft}{\sec^2}\right)}$$

$$time = 22.6 \text{ sec}$$

To calculate the revised takeoff distance using the same relationship as before (recall the area for a right triangle), the distance should be:

$$area = \tfrac{1}{2}(base \% height) = takeoff\ distance$$

$$area = \tfrac{1}{2}[(22.6\sec)(152.1\tfrac{ft}{\sec})]$$

$$takeoff\ distance = 1718.7 \text{ feet}$$

A subsequent takeoff test revealed the takeoff distance to be approximately 1750 feet so the theory appears correct.

Figure 6.5 Takeoff distance determination from chase aircraft

9.0 Summary

Takeoff performance is mainly concerned with the distance required to accelerate the aircraft to a speed where the lift just begins to exceed the weight. The weight, drag and thrust of the aircraft are used in the $F = ma$ equation to determine the acceleration rate. Neglecting the rolling friction yields an acceleration rate which is too high, since the friction acts as a drag force. Assuming the calculated acceleration rate is constant, it is used as the slope of a line on a graph of speed versus time. The lift equation determines the takeoff speed and the time required to accelerate to that speed is found by intersecting the acceleration line at that speed and dropping down to the "time" axis. The area of the triangle formed by this procedure is equal to the takeoff distance.

Warning:

In all of the above calculations, there has been more thrust available than required. In other words, there is excess thrust available. This excess thrust is used to accelerate the aircraft above takeoff speed and is also used to allow the aircraft to climb.

Once the aircraft finally becomes airborne, it begins to climb to altitude. The forces involved in climbs and descents are the subject of the next session.

10.0 Measures of Performance

1 Define "takeoff speed."

2 Why do engineers use 70% of the predicted takeoff speed to determine the aerodynamic drag during the takeoff?

3 Why is the acceleration rate the slope of a line on the graph of velocity versus time?

4 Why does rolling friction increase the takeoff distance?

5 If an aircraft only has enough thrust to accelerate it to takeoff speed, what is the consequence?

11.0 Example

***Problem*:**

Determine the takeoff distance for an aircraft with the following characteristics:

Weight (w): 18,500 lbs
Thrust (T): 6000 lbs
C_{Lmax}: 1.15
C_D: 0.05
Wing area (S): 342 ft^2
Coefficient of friction (μ): 0.06
Air density (ρ): 0.0023 slugs/ft^3

Solution:

Step 1: Determine the takeoff speed.

$$w = L = \tfrac{1}{2}\rho V^2 S C_L \qquad (6.1)$$

$$V = \sqrt{\frac{2w}{\rho S C_L}}$$

$$V = \sqrt{\frac{2(18{,}500 lbs)}{(0.0023\frac{slugs}{ft^3})(342 ft^2)(1.15)}}$$

$$V = 202.2 \text{ ft/sec (or 119.7 knots)}$$

Step 2: Determine the drag by using the drag equation and 70% of the takeoff speed calculated in step 1.

$$V = 0.7\ (202.2 \text{ ft/sec})$$

$$V = 141.5 \text{ ft/sec}$$

$$D = \tfrac{1}{2}\rho V^2 S C_D \qquad (6.2)$$

$$D = \tfrac{1}{2}\left(0.0023\tfrac{slugs}{ft^3}\right)\left(141.5\tfrac{ft}{\text{sec}}\right)^2(342 ft^2)(0.05)$$

$$D = 393.7 \text{ lbs}$$

Step 3: Determine the friction force based on the aircraft weight and a coefficient of friction of 0.06.

$$F_{friction} = (\mu)\ w$$

$$F_{friction} = (0.06)\ (18{,}500 \text{ lbs})$$

$$F_{friction} = 1100 \text{ lbs}$$

Step 4: Determine the acceleration by subtracting the forces calculated in steps 2 and 3 from the total thrust.

$$F = ma$$

$$F = T - D - F_{friction} = \frac{w}{g}a$$

$$a = \frac{g}{w}F = (T - D - F_{friction})$$

$$\boldsymbol{a} = \frac{\left(32.2\frac{ft}{\text{sec}^2}\right)}{(18{,}500 lbs)}(6000 lbs - 393.7 lbs - 1100 lbs)$$

$$a = 7.84 \text{ ft/sec}^2$$

Step 5: Determine the time required to accelerate to the takeoff speed.

$$time = \frac{\text{takeoff speed}}{\text{acceleration rate}}$$

$$time = \frac{202.2\frac{ft}{\text{sec}}}{7.84\frac{ft}{\text{sec}^2}}$$

$$time = 25.79 \text{ sec}$$

Step 6: Determine the takeoff distance assuming the acceleration calculated in step 5 is a constant.

NOTE:

This assumption results in a straight line slope which permits the use of the right triangle formula to calculate the area under the curve.

$$area = \tfrac{1}{2}(base \times height)$$

$$\begin{aligned} &takeoff\ distance \\ &= \tfrac{1}{2}(takeoff\ speed \times time\ to\ \mathbf{r}each\ takeoff\ speed) \end{aligned}$$

$$area = \tfrac{1}{2}\{(25.79\ \mathrm{sec}) \times (202.2\tfrac{ft}{\mathrm{sec}})\}$$

$$area = takeoff\ distance = 2607.5\ ft$$

Problem 1: An aircraft with the following characteristics is operated from a runway 3000 feet long. Can the aircraft takeoff ?

Weight (w): 1800 lbs
Thrust (T): 850 lbs
$C_{L\,max}$: 1.21
C_D: 0.05
Wing area (S): 48 ft^2
Coefficient of friction (μ): 0.03
Air density (ρ): 0.0021 slugs/ft^3

Problem 2: How much thrust is required to takeoff on a 3500 foot runway if an aircraft has the following characteristics:

Weight (w): 3500 lbs
Thrust (T): **?** lbs
C_L: 1.21
C_D: 0.05
Wing area (S): 90 ft^2
Coefficient of friction (μ): 0.03
Air density (ρ): 0.0021 slugs/ft^3